Frequency Data Modeling of Passenger Transport Auto Insurance Claims Using the New Poisson Mixed Weighted Lindley Distribution

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ABSTRAK

Asuransi kendaraan merupakan salah satu instrumen penting dalam pengelolaan risiko. Namun, pemodelan frekuensi klaim sering menghadapi masalah overdispersi sehingga asumsi equidispersi pada distribusi Poisson menjadi tidak terpenuhi. Distribusi alternatif seperti Binomial Negatif telah banyak digunakan untuk mengatasi permasalahan ini, tetapi masih memiliki keterbatasan dalam menangkap heterogenitas klaim pada beberapa data asuransi. Penelitian ini menerapkan distribusi New Poisson Mixed Weighted Lindley (NPWL) pada data frekuensi klaim asuransi mobil angkutan penumpang di Indonesia yang bersumber dari PT. XYZ pada tahun underwriting 2013. Estimasi parameter dilakukan menggunakan pendekatan Maximum Likelihood, dan kecocokan model dievaluasi melalui uji Chi-Kuadrat. Hasil penelitian menunjukkan bahwa model NPWL memberikan kecocokan yang baik terhadap data, dengan nilai statistik uji Chi-Kuadrat sebesar 0,7341 yang lebih kecil dari nilai kritisnya. Estimasi parameter diperoleh sebesar $\widehat{\theta} = 4{,}5919$ dan $\widehat{\alpha} = 0{,}8539$ yang menghasilkan nilai rata-rata 0,3811 dan varians 0,4033. Varians yang lebih besar dari rata-rata menunjukkan bahwa NPWL mampu menangkap overdispersi secara lebih fleksibel dibandingkan model Poisson konvensional, sehingga lebih relevan untuk aplikasi praktis dalam penetapan premi asuransi kendaraan.
Kata kunci: Asuransi Mobil, Frekuensi Klaim, Distribusi NPWL, Overdispersi.

ABSTRACT

Vehicle insurance is an important instrument in risk management. However, claim frequency modeling often faces overdispersion issues, rendering the equidispersion assumption in Poisson distribution invalid. Alternative distributions such as Negative Binomial have been widely used to address this issue, but they still have limitations in capturing claim heterogeneity in some insurance data. This study applies the New Poisson Mixed Weighted Lindley (NPWL) distribution to passenger transport vehicle insurance claim frequency data in Indonesia sourced from PT. XYZ in the 2013 underwriting year. Parameter estimation was performed using the Maximum Likelihood approach, and model fit was evaluated using the Chi-Square test. The results show that the NPWL model provides a good fit to the data, with a Chi-Square test statistic value of 0.7341, which is smaller than the critical value. The parameter estimates obtained were $\widehat{\theta} = 4.5919$ *and* $\widehat{\alpha} = 0.8539$*, which resulted in a mean value of 0.3811 and a variance of 0.4033. The variance being greater than the mean indicates that NPWL is able to capture overdispersion more flexibly than the conventional Poisson model, making it more relevant for practical applications in setting vehicle insurance premiums.*
Keywords: *Car Insurance, Claim Frequency, NPWL Distribution, Overdispersion.*

INTRODUCTION

Insurance plays a crucial role in financial risk management by transferring potential financial losses from individuals to insurance institutions, particularly in the motor vehicle sector, which is characterized by a high level of exposure to accidents and operational uncertainty [1][2], [3]. In Indonesia, the rapid growth in vehicle ownership has increased the importance of accurate risk assessment in motor vehicle insurance, especially for premium determination and loss control. According to data from the Financial Services Authority (OJK), gross premiums in the motor vehicle insurance sector reached IDR 9.39 trillion in 2023, reflecting the growing significance of this line of business [4][5].

One of the main challenges in insurance risk modeling is claim frequency analysis, defined as the number of claims submitted by policyholders within a specific period. Claim frequency is commonly modeled using the Poisson distribution; however, this model relies on the equidispersion assumption, where the mean equals the variance. In practice, insurance claim data often violate this assumption due to unobserved heterogeneity among policyholders, resulting in overdispersion, where the variance exceeds the mean [6][7][8]. Ignoring overdispersion can result in biased premium calculations, potentially harming the insurer or unfairly burdening policyholders.

To address this issue, several mixed Poisson models have been proposed, including the Negative Binomial, Poisson–Gamma, Poisson–Inverse Gaussian, and Poisson–Lindley distributions [9][10][11]. Although these models offer greater flexibility than the standard Poisson distribution, they may still be limited in capturing complex heterogeneity and tail behavior in insurance claim data. The New Poisson Mixed Weighted Lindley (NPWL) distribution extends existing mixed Poisson models by incorporating the Weighted Lindley distribution as a mixing distribution, providing additional flexibility in modeling overdispersion and variability in claim frequencies [12]. However, to the best of the author's knowledge, the application of the NPWL distribution to passenger vehicle insurance claim data in Indonesia has not yet been explored.

This study applies the NPWL distribution to passenger car insurance claim frequency data obtained from PT. XYZ in North Sumatra Province for the 2013 underwriting year. The objectives of this study are threefold: (1) to estimate the parameters of the NPWL distribution, (2) to evaluate the goodness-of-fit of the model to insurance claim frequency data, and (3) to analyze the mean and variance implied by the estimated model. The findings are expected to contribute theoretically by providing an empirical application of the NPWL distribution in insurance modeling and practically by supporting more accurate premium determination and risk classification in passenger vehicle insurance.

METHOD

Research Data

This study employs a quantitative approach to model passenger vehicle insurance claim frequency using the New Poisson Mixed Weighted Lindley (NPWL) distribution. The data consist of secondary claim frequency records obtained from PT. XYZ in North Sumatra Province for the 2013 underwriting year. The sample comprises 137 passenger vehicle insurance policies, selected based on an inclusion criterion of insured values ranging from Rp200.000.000 to Rp400.000.000. Policies outside this coverage range, as well as records with incomplete claim information, were excluded from the analysis.

The selected coverage interval represents a relatively homogeneous risk segment commonly used in motor insurance underwriting. This allows claim frequency behavior to be analyzed without distortion from extreme insured values. Focusing on this segment ensures that the estimated NPWL parameters accurately reflect claim variability within a comparable risk class, thereby enhancing the relevance of the model for premium determination and risk assessment.

Tabel 1. Claim Frequency Data

Number of Claims	Number of Policies
0	97
1	31
2	6
3	3
Total	**137**

Overdispersion Test

To identify overdispersion in claim frequency data, the first step is to compare the sample mean with its variance. Overdispersion can be seen if the variance of the data is higher than its mean, which contradicts the assumption of equidispersion in the Poisson distribution. One statistical method often used to detect overdispersion is the Variance Test (VT) . The hypothesis of this variance test is determined as follows:

H_0: The data do not exhibit overdispersion
H_1: The data exhibit overdispersion

the VT test statistic is given by:

$$VT = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\bar{X}} = (n-1)\frac{S^2}{\bar{X}} \quad (1)$$

Where n denotes the sample size, S^2 is the sample variance, and $\bar{X}$ is the sample mean. The VT statistic follows a chi-square distribution with (n − 1) degrees of freedom. The null hypothesis is rejected when VT exceeds the corresponding chi-square critical value. In this study, a significance level of α = 10% is adopted to increase sensitivity in detecting overdispersion, which is commonly recommended for exploratory insurance claim data with moderate sample sizes.

New Poisson Mixed Weighted Lindley Distribution

Claim frequency data exhibiting overdispersion are modeled using the New Poisson Mixed Weighted Lindley (NPWL) distribution. The NPWL distribution is a mixed Poisson model in which the Poisson rate parameter follows a modified Weighted Lindley distribution [13].

The probability mass function of the Poisson distribution is given by:

$$P(X = x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots \quad (2)$$

The Weighted Lindley distribution, used as the mixing distribution is defined by probability density function:

$$f(\lambda) = \frac{\theta^2}{1+\theta}(1+\lambda)e^{-\theta\lambda},\ \lambda > 0, \theta > 0 \tag{3}$$

By mixing the Poisson distribution with the modified Weighted Lindley distribution, the probability mass function of the NPWL distribution is obtained as:

$$f(x) = \frac{\theta^2(1+\alpha)^2}{\alpha\theta(1+\alpha)+\alpha}\left(\frac{\theta+x+2}{(\theta+1)^{x+2}} - \frac{\alpha\theta+\theta+x+2}{(\alpha\theta+\theta+1)^{x+2}}\right) \tag{4}$$

for $x = 0, 1, 2, \ldots$, where $\theta > 0$ and $\alpha > 0$.

In this model, θ represents the baseline claim intensity, while α captures unobserved heterogeneity among policyholders. The inclusion of α provides additional flexibility, allowing the NPWL distribution to effectively accommodate overdispersion commonly observed in insurance claim frequency data. Detailed mathematical derivations are provided in the Appendix.

Moments of the NPWL Distribution

The first and second moments of the NPWL distribution are expressed as:

$$E(X) = \delta_1\delta_3 \tag{5}$$

$$E(X^2) = \delta_1\left(\frac{\theta^2+4\theta+6}{\theta^4} - \frac{\delta_2^2+4\delta_2+6}{\delta_2^4}\right) \tag{6}$$

where:

$$\delta_1 = \frac{\theta^2(1+\alpha)^2}{\alpha\theta(1+\alpha)+\alpha(2+\alpha)} \tag{7}$$

$$\delta_2 = \alpha\theta + \theta \tag{8}$$

$$\delta_3 = \frac{\theta+2}{\theta^3} - \frac{\alpha\theta+\theta+2}{(\alpha\theta+\theta)^3} \tag{9}$$

The variance of the NPWL distribution is obtained as:

$$Var(X) = \delta_1\left(\frac{\theta^2+4\theta+6}{\theta^4} - \frac{\delta_2^2+4\delta_2+6}{\delta_2^4}\right) - \delta_1^2\delta_3^2 \tag{10}$$

Parameter Estimation

The parameters estimation of the NPWL distribution are estimated using the Maximum Likelihood Estimation (MLE) method. Let $x_1, x_2, \ldots, x_n$ denoted a random sample from the NPWL distribution. The likelihood function is defined as:

$$L(\alpha,\theta) = \prod_{i=1}^{n}\{f(x_i)\}$$

$$= \prod_{i=1}^{n}\left\{\frac{\theta^2(1+\alpha)^2}{\alpha\theta(1+\alpha)+\alpha}\left(\frac{\theta+x_i+2}{(\theta+1)^{x_i+2}} - \frac{\alpha\theta+\theta+x_i+2}{(\alpha\theta+\theta+1)^{x_i+2}}\right)\right\} \tag{11}$$

and the corresponding log-likelihood function is given by:

$$l(\alpha,\theta) = \ln L(\alpha,\theta)$$
$$= 2n\ln(\theta) + 2n\ln(1+\alpha) - n\ln(\alpha) - n\ln(\theta(1+\alpha)+2+\alpha)$$
$$-(\textstyle\sum_{i=1}^{n} x_i + 2n)\log(\theta+1) - (\textstyle\sum_{i=1}^{n} x_i + 2n)\ln(\alpha\theta+\theta+1)$$
$$+\sum_{i=1}^{n}\ln((\theta+x_i+2)(\alpha\theta+\theta+1)^{x_i+2}(\alpha\theta+\theta+x_i+2)(\theta+1)^{x_i+2}) \quad (12)$$

Due to the highly nonlinear from of the log-likelihood function, closed-from solutions for the parameter estimates cannot be obtained analytically [14]. There fore, the Newton-Raphson numerical optimization method is employed.

Let $\gamma = (\alpha,\theta)^T$ denote the parameter vector. The Newton-Raphson iterative scheme is given by:

$$\gamma^{(t+1)} = \gamma^{(t)} - \left[\frac{\partial^2 l(\gamma)}{\partial\gamma\partial\gamma^T}\bigg|_{\gamma=\gamma^{(t)}}\right]^{-1}\left[\frac{\partial l(\gamma)}{\partial\gamma}\bigg|_{\gamma=\gamma^{(t)}}\right], t = 0,1,2,\ldots \quad (13)$$

The iteration process is terminated when the convergence criterion $\left|\gamma^{(t+1)} - \gamma^{t}\right| < \varepsilon$, where $\varepsilon = 10^{-6}$. All numerical computations and parameter estimations are carried out using R Software

Chi-Square Goodness-of-Fit Test

The distribution fit test is a statistical hypothesis testing method that aims to determine whether the data $x_1, x_2, \ldots, x_n$ are values from a random sample $X_1, X_2, \ldots, X_n$ taken from a distribution with the distribution function $F(\cdot)$. In the distribution fit test, the hypotheses that can be used are as follows [15]:

H_0: $x_1, x_2, \ldots, x_n$ is the value of a random sample distributed with the distribution function $F(\cdot)$.
H_1: $x_1, x_2, \ldots, x_n$ is the value of a random sample distributed with a non-uniform distribution function $F(\cdot)$.

The test statistic used to test the goodness of fit of chi-square on discrete data is:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \quad (14)$$

where:
O_i: number of observations in a category *i*
E_i: expected value in the category *i.*

to get the value of E_i it can be calculated using the following equation:

$$E_i = nP_x, x = 0,1,2,\ldots \quad (15)$$

The critical value is obtained from the chi-square distribution with $k - p - 1$ degrees of freedom, where k is the number of categories and p is the number of estimated parameters [16]. The null hypothesis is rejected if the computed chi-square statistic exceeds the critical value at a significance level of $\alpha = 5\%$ and $k - p - 1$ degrees of freedom atau $X^2 \geq X^2_{(k-p-1)(1-\alpha)}$.

RESULT AND DISCUSSION

Data Description

This study uses a quantitative approach to develop a passenger vehicle insurance claim frequency model using the New Poisson Mixed Weighted Lindley (NPWL) distribution. The data used are secondary claim frequency records obtained from PT. XYZ in North Sumatra Province for the 2013 offering year. The sample consisted of 137 passenger vehicle insurance policies, selected based on inclusion criteria with insured values between Rp200,000,000 and Rp400,000,000. Policies outside this coverage range and records with incomplete claim information were excluded from the analysis.

The selected coverage interval reflects a uniform risk segment typically used in motor vehicle insurance underwriting, allowing claim frequency behavior to be analyzed without interference from extreme insured values. This focus ensures that the estimated NPWL parameters reflect claim variability within comparable risk classes, making the model results more relevant for premium determination and risk assessment.

Overdispersion Test

To formally assess the presence of overdispersion, a Variance Test (VT) was conducted. Table 3 presents a brief summary of the test results. The VT statistic exceeded the critical chi-square value associated with a 10% significance level, resulting in the rejection of the null hypothesis of dispersion equality.

The use of a 10% significance level is considered appropriate in this context, as insurance claim data is typically exploratory and involves moderate sample sizes. This choice increases the sensitivity of the test in detecting overdispersion, which is crucial for selecting the appropriate frequency model.

Tabel 2. Overdispersion Test Summary

Statistic	Value
Sampel Size (n)	137
Sampel Mean	0,3796
Sampel Variance	0,4545
VT Statistic	162,8346
X^2critical value (α = 10%, df = 136)	157,518
Conclusion	Overdispersion detected

Parameter Estimation of the NPWL Distribution

The parameters of the NPWL distribution are estimated using the Maximum Likelihood Estimation (MLE) method. Due to the complexity of the log-likelihood function, closed-form solutions cannot be obtained analytically. Therefore, the Newton–Raphson iterative method is employed.

The initial values for the parameters are selected based on reasonable starting points to ensure convergence. The iteration process is stopped when the absolute difference between successive parameter estimates is less than ε = 1 x 10^{-6}.

Tabel 3. NPWL Parameter Estimation

Iteration	θ	α	Log-likelihood
Initial parameter values	4,722257	0,7233883	
Iteration 1	4,5931	0,8525	-110,8272
Iteration 2	4,5919	0,8539	-110,8272
Result	4,5919	0,8539	-110,8272

NPWL Distribution Suitability Test Results

Testing the suitability of the NPWL distribution on the claim frequency data of passenger car insurance policyholders with a coverage price of more than Rp200,000,000 to Rp400,000,000 from PT. XYZ in North Sumatra Province in the underwriting year 2013 using a chi-square goodness-of-fit test. The following is the hypothesis for the test:

H_0: The frequency data of passenger car insurance claims with coverage value of more than Rp200.000.000 to Rp400.000.000 from PT. XYZ in North Sumatra Province in the underwriting year 2013 comes from the population distributed by NPWL.

H_1: The frequency data of passenger car insurance claims with coverage value of more than Rp200.000.000 to Rp400.000.000 from PT. XYZ in North Sumatra Province in the underwriting year 2013 did not come from the population distributed by NPWL.

To meet the assumptions of the chi-square test, claim frequencies of three or more were combined into a single category due to low expected frequencies. The estimated NPWL parameters obtained via maximum likelihood estimation are $\hat{\theta}$ = 4,5919 dan $\hat{\alpha}$ = 0,8539.

Tabel 4. Values in the calculation of test statistics

Category (i)	Claim Frequency (x)	Number of Policies (O_i)	Claim Frequency Opportunity (P_x)	Expected Value of Claim Frequency (E_i)	$\frac{(O_i - E_i)^2}{E_i}$
1	0	97	0,7066	96,8067	0,0004
2	1	31	0,2243	30,7328	0,0023
3	2	6	0,0542	7,4255	0,2737
4	≥3	3	0,0149	2,0349	0,4577
Total		**137**	**1**	**137**	**0,7341**

then calculate the chi-square test statistic value. The chi-square test statistic value is 0.7341. With a significance level of α = 10% the critical value of the chi-square distribution with degrees of freedom 1 (= 4 – 2 – 1) is 2,076. It can be seen that the test statistic value is smaller than the critical value, therefore, H_0 is accepted and it is concluded that the frequency data of passenger car insurance claims with a coverage price of more than Rp200.000.000 to Rp400.000.000 from PT. XYZ in North Sumatra Province in the underwriting year 2013 comes from a population distributed with NPWL with parameter estimates $\hat{\theta}$ = 4,5919 dan $\hat{\alpha}$ = 0,8539. To satisfy the requirement of the chi-square goodness-of-fit test, categories with low expected frequencies are combined. In this study, claim frequencies of three or more (x ≥ 3) are grouped into a single category to ensure that all expected values are sufficiently large for valid statistical inference.

Estimated Mean and Variance of Claim Frequency

Estimated average value of NPWL distribution based on the estimated parameters $\hat{\theta}$ = 4,5919 and $\hat{\alpha}$ = 0,8539. The following are the results of the calculation of the estimated average value of the frequency of passenger car insurance claims in North Sumatra in the 2013 underwriting year.

$$\hat{E}(X) = \widehat{\delta_1 \delta_3} = \frac{\theta^2(1+\alpha)^2}{\alpha\theta(1+\alpha)+\alpha(2+\alpha)} \cdot \frac{\theta+2}{\theta^3} - \frac{\alpha\theta+\theta+2}{(\alpha\theta+\theta)^3}$$
$$= \frac{4{,}5919^2(1+0{,}8539)^2}{0{,}8539 \cdot 4{,}5919(1+0{,}8539)+0{,}8539(2+0{,}8539)} \cdot \frac{4{,}5919+2}{4{,}5919^3} - \frac{0{,}8539 \cdot 4{,}5919+4{,}5919+2}{(0{,}8539 \cdot 4{,}5919+4{,}5919)^3}$$
$$= 0{,}3811$$

Based on the analysis results, the estimated average value of the NPWL distribution for the frequency of claims for category 3 passenger car insurance in North Sumatra in the 2013 underwriting year was 0,3811. This value indicates that for every 10.000 policyholders, an estimated 3,811 claims occurred in a year.

Next, the variance estimate of the NPWL distribution is calculated using the formula:

$$\hat{V}ar(X) = \delta_1\left(\frac{\theta^2+4\theta+6}{\theta^4} - \frac{\delta_2^2+4\delta_2+6}{\delta_2^4}\right) - \delta_1^2\delta_3^2$$

with:

$$\widehat{\delta_2} = \alpha\theta + \theta$$
$$= 0{,}8539\theta \cdot 4{,}5919 + 4{,}5919$$
$$= 0{,}8539 \cdot 4{,}5919 + 4{,}5919 = 8{,}5129$$

$$\hat{E}(X^2) = \widehat{\delta_1}\left(\frac{\theta^2+4\theta+6}{\theta^4} - \frac{\delta_2^2+4\delta_2+6}{\delta_2^4}\right)$$
$$= \frac{\theta^2(1+\alpha)^2}{\alpha\theta(1+\alpha)+\alpha(2+\alpha)} \cdot \left(\frac{\theta^2+4\theta+6}{\theta^4} - \frac{\delta_2^2+4\delta_2+6}{\delta_2^4}\right)$$
$$= \frac{4{,}5919^2(1+0{,}8539)^2}{0{,}8539 * 4{,}5919(1+0{,}8539)+0{,}8539(2+0{,}8539)} \cdot \left(\frac{4{,}5919^2+4 \cdot 4{,}5919+6}{4{,}5919^4} - \frac{8{,}5129^2+4 \cdot 8{,}5129+6}{8{,}5129^4}\right)$$
$$= 0{,}5485$$

so that the variance value:

$$\hat{V}ar(x) = \delta_1\left(\frac{\theta^2+4\theta+6}{\theta^4} - \frac{\delta_2^2+4\delta_2+6}{\delta_2^4}\right) - \delta_1^2\delta_3^2$$

$= 0{,}5485 - 0{,}3811^2$
$= 0{,}4033$

Based on the calculation results, the variance value of the NPWL distribution is 0.4033. It can be seen that the estimated variance value of the NPWL distribution is greater than the estimated mean value of the NPWL distribution. This proves that the NPWL distribution is an alternative distribution that can overcome the problem of overdispersion in the data.

CONCLUSION

This study shows that the frequency data of passenger car insurance claims with coverage values between Rp200,000,000 and Rp400,000,000 in North Sumatra Province for the 2013 insurance year shows overdispersion, as indicated by a variance that is greater than the average. The estimated NPWL parameters confirm the existence of heterogeneity among policyholders, while the chi-square goodness-of-fit test provides empirical evidence that the NPWL distribution fits the observed data well.

Compared to the standard Poisson model that assumes equidispersion, the NPWL distribution offers greater flexibility by explicitly accounting for unobserved heterogeneity, resulting in a more realistic representation of claim frequency variability. This improvement has practical implications for insurance applications, particularly in supporting more accurate premium setting and risk classification for passenger vehicle insurance.

This study has several limitations, including the use of data from a single year of coverage, a single insurance company, and a relatively small maximum claim frequency. Future research may address these limitations by incorporating multi-year datasets, expanding the model to a regression framework with policyholder-specific covariates, or conducting comparative analyses with other mixed Poisson distributions to further evaluate the practical benefits of the NPWL model.

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