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Three–Phase Traffic Light Petri Net Model Using The Modified Norwegian System

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ABSTRAK

Petri net dapat dipakai untuk memodelkan perilaku sinyal lampu lalu lintas. Model Petri net juga memungkinkan untuk menyajikan singkronisasi beberapa fase lampu lalu lintas. Model Petri net dapat mereprentasikan sinyal modifikasi sistem Norwegia yang menyala dengan urutan hijau, kuning, merah, kuning, dan kemudian hijau lagi. Pada setiap siklus lampu lalu lintas modifikasi sistem Norwegia, sinyal kuning menyala dua kali. Studi ini bertujuan untuk mengkaji model Petri net lampu lalu lintas dengan tiga fase yang menggunakan modifikasi sistem Norwegia. Metode uji validasi dan verifikasi kebenaran model Petri net diantaranya menggunakan sejumlah *Place-Invariant*, properti keterbatasan (*boundedness*) pada Petri net, konservasi (*conservation*), *coverability tree* untuk berbagai keadaan sinyal, dan simulasi. Berdasarkan hasil kajian menyatakan bahwa *Place-Invariant*, dan properti Petri net dapat merepresentasikan bahwa model adalah benar dan *feasible*. Simulasinya juga menyajikan urutan yang benar sinyal modifikasi sistem Norwegia.

Kata kunci: model Petri net, lampu lalu lintas, modifikasi sistem Norwegia

ABSTRACT

Petri nets can be used to model the behavior of traffic light signals. The Petri net model also makes it possible to provide synchronization of several traffic light phases. The Petri net model can represent the modified Norwegian system signal, which lights up in the sequence green, yellow, red, yellow, and then goes back to green again. The yellow signal flashes twice in each traffic light cycle modified Norwegian system. This study aims to examine the Petri net model of traffic lights with three phases using the modified Norwegian system. Methods for validating and verifying the correctness of the Petri net model used several Place-Invariants, boundedness properties on the Petri net, conservation, coverability trees for various signal conditions, and simulation. Based on the study results, the Place-Invariant and the Petri net properties can represent that the model is correct and feasible. The simulation also presents the correct sequence of modified Norwegian system signals.

Keywords: *Petri net model, traffic lights, modified Norwegian system*

INTRODUCTION

This study aims to study the Petri net model of the modified Norwegian traffic light system. Traffic lights have three phases and a fixed order: west, north, and east. They then return to the initial state, the west phase.

Generally, a standard traffic light system has three discrete state sequences: green, yellow, and red. All states light up in specific and at permanent time intervals, alternating with other phases and repeatedly forming a traffic light cycle [1]. The Norwegian traffic light system is slightly different from the standard system. On the Norwegian system, the yellow and red signals flash simultaneously before returning to the green signal. These two signals aim to reduce travel delays and not to trigger conflict. If the signal lights up red–yellow, vehicles stopped for a long time can prepare to start their travel again. The Modified Norwegian system changes the red–yellow signal to a single-yellow signal. In each traffic light cycle of the modified Norwegian system, the yellow signal flashes twice [2].

Petri nets are models that can graphically represent the behavioral structure of a distributed system. It can also model control systems, sensor networks, and manufacturing [3]. Petri nets can also represent traffic light scheduling [2]. Petri net is a directed graph with four essential elements: places (P/roundabouts), transitions (T/squares), directed arcs, and tokens/dots. Place (P) is a representation of the state that occurred. The enabled transition (T) is ready to fire. It can trigger and transform an initial state into the next state according to the arc direction. A token indicates that a condition related to a particular place is occurring [4].

Several Petri net properties, including boundedness, conservation, and state tree coverability, are used for validation tests and model verification. Model validation and verification tests also use Place-Invariant and simulation. The analysis uses the coverability tree method, a sequence of state occurred, and fire transitions, which can be represented using the multiplication of the connection/ incidence matrix and the enable transition matrix. [3].

Previous research has studied the structure of traffic light behavior using Petri nets. It is the study of traffic light structure modifications using Petri nets, which can reduce delays but do not trigger conflicts [2]. The study of invariants on Petri nets was conducted to prove that a traffic light model is correct [4]. Research on modified binary Petri nets (MBPNs) to design traffic light models [5]. Research on the validity of traffic light models and analysis using Petri net properties [6]. Business process system studies must be deadlock-free. This study was analyzed using the Petri net model [7]. It is the research on the Petri net model of a network system with multichannel queues [8].

METHOD

The following describes the definition of a Petri net, the three-phase system using the modified Norwegian system, and the connectivity/incidence matrix and invariants.

1. Petri Net

Four Petri net elements must be known, including N(P, T, A, w). The elements of P are a finite set of places. $P = \{P_1, P_2, \ldots, Pm\}$. The finite set of transitions T is $T = \{t_1, t_2, \ldots, tn\}$, which m and n are positive integers. Element A is a set of directed arcs that relate place to transition and the reverse, namely from transition to place. It is written as $A \subseteq (P\times T) \cup (T\times P)$. The element w (weight) is the weight function of an arc with w, i.e., A→{1, 2, 3, . . }. The model's arc weight is written near the arc line using an integer number that is greater than one. It is presented with a directed line only if it equals one [3].

The concept of 'marking' in a Petri net model refers to the use of tokens in a place, which is mathematically represented as M(Pi) ={0, 1}, where i=1,2,3,..15. There are 15 places in a traffic light model with three phases of the modified Norwegian system. A 'marking place' in a traffic light signal model represents the states marked with tokens. For a signal to be on, its corresponding place will have a token, which will be empty if the signal is off. The symbol '1' represents 'on ', while '0' represents 'off '.

2. Three–Phase Traffic Light Using Modified Norwegian System

Figure 1., shows a road junction with a three-phase traffic light schedule. Phase scenario is a traffic light scheduling design that aims to ensure that the space at a road intersection can be used alternately by a group of vehicles coming from different directions [9].

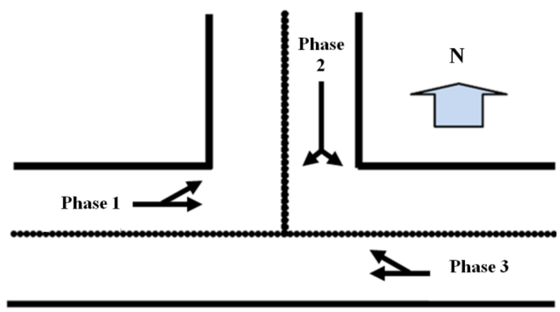

Figure 1. Road junction with three arms and its traffic flow

Figure 2. presents a three-phase traffic light Petri net model with a modified Norwegian system. The west arm features signals P_1, P_2, and P_3, representing green, yellow, and red signals. The north arm has P_6, P_7, and P_8, and the east has P_{11}, P_{12}, and P_{13} signals. Control places P_4, P_9, and P_{14} are crucial in the system. They are responsible for changing the yellow signal to red. Similarly, control places P_5, P_{10}, and P_{15} facilitate transforming the red signal to yellow. These three places also function to synchronize the three traffic light phases.

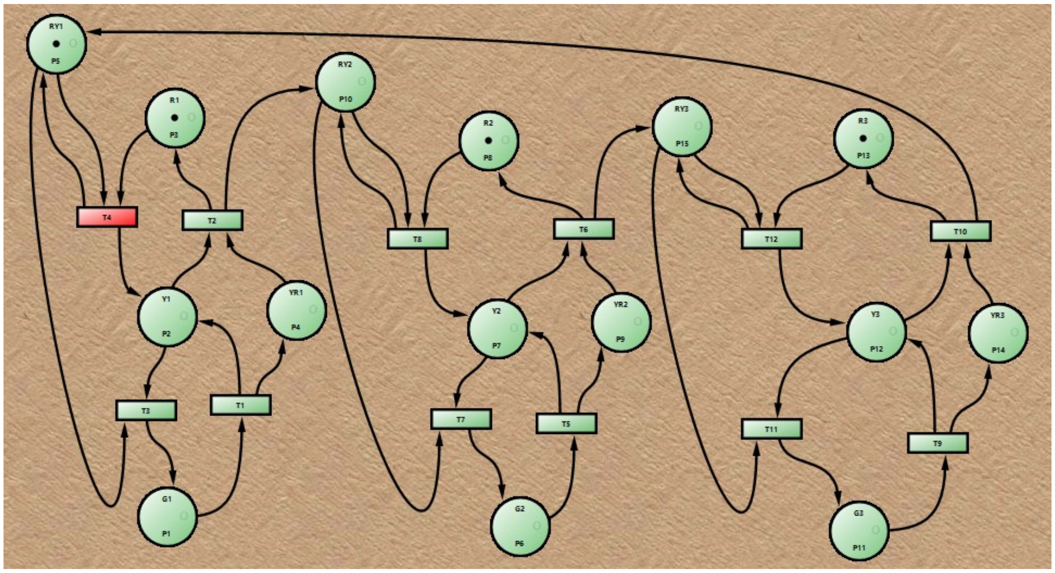

Figure 2. Petri net model of three-phase traffic lights with a modified Norwegian system when the All Red Signal is on.

As depicted in Figure 2., a token on one of P_5, P_{10}, or P_{15} triggers the activation of All Red signals, causing all traffic light signals for the three arms to turn red. There is always an All Red Signal at the end of each traffic light phase of an arm. The aim is to provide a moment's pause to clear the junction of other vehicles that have not yet finished passing, thereby ensuring travel safety and preventing traffic flow conflicts [2]. The synchronous three-phase traffic lights are preceded by the T_2, T_6, or T_{10} fire transition and then progress with T_8, T_{12}, or T_4, respectively.

The green signal for the west arm is on if there is a token on P1. The signals on the north and east arms must be red. There should be a token each in places P_8 and P_{13}. It is illustrated in Figure 2. The signal will light up yellow after the red light elapses and then transform to a green signal in phase 1, namely phase west.

3. The Connectivity/ Incidence Matrix and Invariants

Connections in the Petri net model can be represented using a Forward Incidence Matrix and a Backward Incidence Matrix. The connectivity matrix of the Petri net model is presented in Figure 3. The connectivity matrix A subtracts the Backward Incidence Matrix from the Forward Incidence Matrix. The matrix size is 15 x 12, meaning the model has 15 places and 12 transitions.

The value of the Forward Incidence Matrix elements is the weight of the arcs that connect the transition to place, or place is the output of the transition. The Backward Incidence Matrix elements are the weight of the arcs that connect the place to the transition or mean that the places are the transitions's input. If there is no arc connecting the place to the transition or vice versa, then the arc weight is given a value of zero [3].

A =

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12		
	-1	0	1	0	0	0	0	0	0	0	0	0	G1	= P1
	1	-1	-1	1	0	0	0	0	0	0	0	0	Y1	= P2
	0	1	0	-1	0	0	0	0	0	0	0	0	R1	= P3
	1	-1	0	0	0	0	0	0	0	0	0	0	YR1	= P4
	0	0	-1	0	0	0	0	0	0	1	0	0	RY1	= P5
	0	0	0	0	-1	0	1	0	0	0	0	0	G2	= P6
	0	0	0	0	1	-1	-1	1	0	0	0	0	Y2	= P7
	0	0	0	0	0	1	0	-1	0	0	0	0	R2	= P8
	0	0	0	0	1	-1	0	0	0	0	0	0	YR2	= P9
	0	1	0	0	0	0	-1	0	0	0	0	0	RY2	= P10
	0	0	0	0	0	0	0	0	-1	0	1	0	G3	= P11
	0	0	0	0	0	0	0	0	1	-1	-1	1	Y3	= P12
	0	0	0	0	0	0	0	0	0	1	0	-1	R3	= P13
	0	0	0	0	0	0	0	0	1	-1	0	0	YR3	= P14
	0	0	0	0	0	1	0	0	0	0	-1	0	RY3	= P15

Figure 3. Connectivity/ incidence matrix of the Petri net model

Invariant, or Place-Invariant, is a term that describes a Linear Time-Invariant (LTI) system in dynamic marking places. Traffic lights are a Linear Time-Invariant (LTI) system. The system has dynamic behavior, but the marking uses tokens; it has permanent elements unaffected by time [10]. Invariants are a property of Petri net used to validate and verify the correctness of a model. While this study does not discuss invariants in transitions, they are also a part of this concept. The manifestations of Invariants or Place-Invariants are written in Invariant (1) to Invariant (3). Extra, Invariant (4), and Invariant (5) are used to present a conservation number of Petri net models.

Traffic lights regulate travel schedules at intersections/ junctions to avoid conflicts in the flow of vehicles coming from different directions. It must serve all signal phases of the road arm and can return to the initial state to create cycles and repetition of traffic light schedules [3].

T =

T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12
1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1

Figure 4. A set of transition matrices in a modified Norwegian traffic light system for junctions with three arms.

In Figure 4, the transition matrix is presented in order from T_1 to T_{12}. However, Figure 4 does not reflect the fire sequence in the traffic light Petri net model with a modified Norwegian system.

RESULT AND DISCUSSION

Figure 5 shows the sequence of events in the Petri net model of three-phase traffic lights, which was designed to implement the modified Norwegian system. Twelve events are combined in a row into a matrix O (Occurrence), which is 15 x 12.

O =

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12
P1	1	0	0	0	0	0	0	0	0	0	0	0
P2	0	1	0	0	0	0	0	0	0	0	0	1
P3	0	0	1	1	1	1	1	1	1	1	1	0
P4	0	1	0	0	0	0	0	0	0	0	0	0
P5	0	0	0	0	0	0	0	0	0	0	1	1
P6	0	0	0	0	1	0	0	0	0	0	0	0
P7	0	0	0	1	0	1	0	0	0	0	0	0
P8	1	1	1	0	0	0	1	1	1	1	1	1
P9	0	0	0	0	0	1	0	0	0	0	0	0
P10	0	0	1	1	0	0	0	0	0	0	0	0
P11	0	0	0	0	0	0	0	0	1	0	0	0
P12	0	0	0	0	0	0	0	1	0	1	0	0
P13	1	1	1	1	1	1	1	0	0	0	1	1
P14	0	0	0	0	0	0	0	0	0	1	0	0
P15	0	0	0	0	0	0	1	1	0	0	0	0

Figure 5. The Occurrence Matrix presents all states/ events in the modified Norwegian traffic light system.

Equations (1) to Equation (7) are determined based on the connection matrix/ Incidence Matrix A shown in Figure 3. It is the formula after fire with the enabled transition. A transition is enabled if all input places contain tokens greater than the arc weight to the next destination place [3].

$$O_{i+1}=O_i+A.T_i\,,\ \text{which}\ \ i=1, 2, 5, 6, 9,10 \quad (1)$$

$$O_4=O_3+A.T_8 \quad (2)$$

$$O_5=O_4+A.T_7 \quad (3)$$

$$O_8=O_7+A.T_{12} \quad (4)$$

$$O_9=O_8+A.T_{11} \quad (5)$$

$$O_{12}=O_{11}+A.T_4 \quad (6)$$

$$O_1=O_{12}+A.T_3 \quad (7)$$

Figure 6 shows the sequence of fire transitions and Occurrences. One traffic light cycle starts from [O1] → T1, with the event [O1] and the transition T1 enabled. All transitions in the model fire once in one traffic light cycle without missing anything, and events can return to their initial state, which indicates coverability.

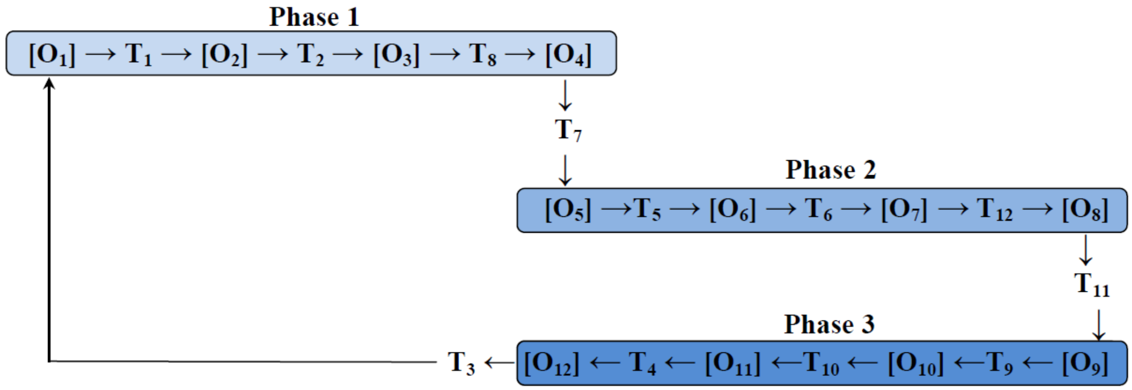

Figure 6. The sequence of all occurrences and fire transitions.

The following are three permanent invariants with parts that do not change over time. Invariant (1) states that only one signal can light at a phase in one junction arm: green, yellow, or red. The *Marking*: $M(G_i), M(Y_i), M(R_i), M(G_j), M(Y_j), M(R_j), M(YR_i), M(RY_i) = \{0,1\}$.

Invariant (2) states that a traffic light signal on a phase may turn on a green or yellow signal if the signals of the other two arms are light red. Invariant (2) guarantees travel safety for vehicles coming from an arm so there is no conflict with other traffic from the different road arms.

$$M(G_i) + M(Y_i) + M(R_i) = 1, \ for\ i, j = 1, 2, 3 \qquad \text{Invariant (1)}$$

$$M(G_i) + M(Y_i) + M(R_i) = M(R_j) \ \ for\ \ i \neq j, and\ \ M(R_j) = 1 \qquad \text{Invariant (2)}$$

$$M(G_1) + M(YR_1) + M(RY_1) + M(G_2) + M(YR_2) + M(RY_2) + M(G_3) + M(YR_3) + M(RY_3) = 1 \qquad \text{Invariant (3)}$$

Invariant (3) describes a synchronized travel schedule for the three road arms of the junction. *M(YRi)* is the marking symbol on the control place for the yellow signal, which will transform into a red signal for i = 1, 2, 3. *M(RYi)* is the marking on the synchronized control place for the three arms and the control place for the red signal, which will transform into a yellow signal at a traffic light phase.

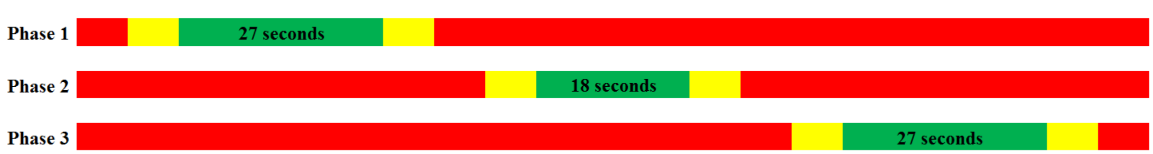

Figure 7. Three-phase traffic light simulation results with a modified Norwegian system

After the verification, the modified Norwegian traffic light system model with three phases was declared valid. The fire sequence of transitions in the traffic light Petri net model, which has three phases, constructs a traffic light cycle. The model has a qualified coverability tree that can return to its initial state.

Table 1. The Traffic Light Schedule

Phase	Green	InterGreen			Red	Cycle
		Yellow I	Yellow II	All Red		
	Seconds					
1. West	27	3	3	3	66	99
2. North	18	3	3	3	75	99
3. East	27	3	3	3	66	99

Figure 7 shows the simulation. Table 1 presents the complete schedule. Both present the correct sequence and time interval for each signal of the modified Norwegian system. The traffic light Petri net model meets the boundedness property; the number of tokens in a place does not grow to infinity or is bounded. The model also fulfills the conservation property. There are no deadlocks or situations when no transitions can fire at all.

$$2M(P_1) + M(P_2) + M(P_3) + M(P_5) + 2M(P_6) \\ +M(P_7) + M(P_8) + M(P_9) + M(P_{10}) + 2M(P_{11}) \\ +M(P_{12}) + M(P_{13}) + M(P_{14}) + M(P_{15}) = 4 \qquad \text{Invariant (4)}$$

Petri net is conservation if the number of tokens in all places after being multiplied by their respective weights is constant for all situations [3]. Multiplication of the matrix B(1x15) on the Occurrence matrix O in Figure 5., will always give a constant value. Multiplication B(1x15) x O(15 x 12) = C(1x12). The row matrix B = [2 1 1 1 2 1 1 1 2 1 1 1 1 1 1] and the row matrix C = [4 4 4 4 4 4 4 4 4 4 4 4]. There are 12 occurrence states in one modified Norwegian traffic light cycle. The conservation constant value is 4. This conservation property can be written in Invariant (4). $M(P_i) = \{0,1\},\ which\ i = 1, 2, \dots 15$.

$$2M(G_1) + M(K_1) + M(R_1) + M(YR_1) + M(RY_1) + 2M(G_2) \\ +M(Y_2) + M(R_2) + M(YR_2) + M(RY_2) + 2M(G_3) \\ +M(Y_3) + M(R_3) + M(YR_3) + M(RY_3) = 4 \qquad \text{Invariant (5)}$$

Invariant (4), which applies to all places in the model, can also be written as Invariant (5). The marking applies: $M(G_i), M(Y_i), M(R_i), M(G_j), M(Y_j), M(R_j), M(YR_i), M(RY_i) = \{0,1\}$. Solving conservation problems is sufficient if the coverability tree is met.

Three-phase traffic lights can be implemented at all road junctions. These lights, designed for safety, are particularly effective at most Trans Java Toll exits – in Indonesia, which are junctions with three arms. When the schedule is appropriate, these lights ensure safe and smooth traffic flow without causing jams.

CONCLUSION

A study has reviewed the traffic light model with three phases of the modified Norwegian system using Petri nets, along with model correctness verification tests using Invariants and several Petri net properties. Based on the study results, the model was declared valid and feasible. The Petri net property can show that the model can return to the initial. Analysis of the traffic light Petri net model has fulfilled the boundedness, conservation, and tree coverability properties for all conditions. The simulation has presented the correct sequence of each signal in the modified Norwegian system.

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